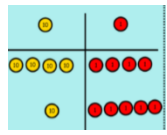
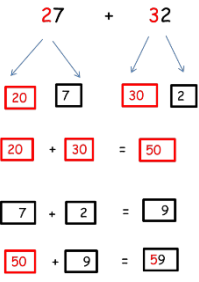
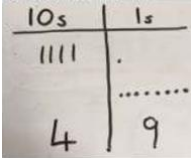
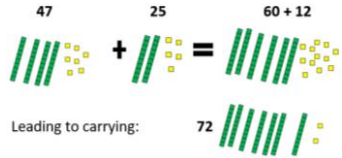
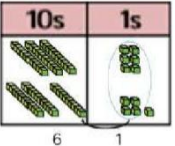
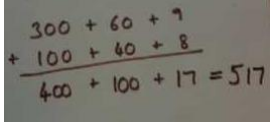
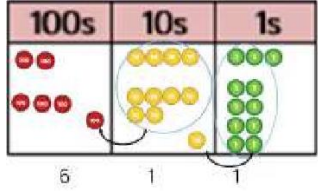
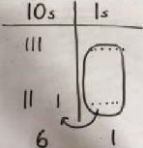
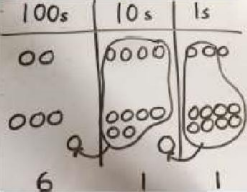
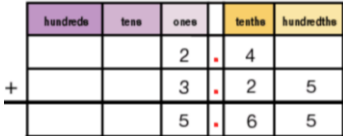


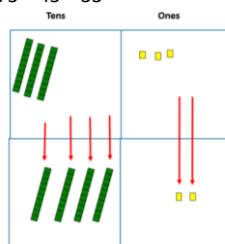
	Concrete	Pictorial	Abstract
Combining two parts (aggregation)	Combine 2 sets Group 2 numbers together and combine in a bar Part – whole model	Use pictures to add 2 numbers together Progress to diagrams or bar model Part – whole model can also be used	Children need to understand the concept of equality before using the '=' sign. Calculations should be written either side of the equality sign so that the sign is not just interpreted as 'the answer'. $5 = 2 + 3$ $1 + 4 = 5$ Missing numbers need to be in all possible places. $3 + 4 = \square$ $3 + \square = 7$ $\square = 3 + 4$ $7 = \square + 4$
Starting at the bigger number and counting on (augmentation)	Count the larger group first then add on 1 by 1 Start with the larger number grouped together then count on to the smaller number to find the answer. $8 + 1 = 9$	Start at the larger number on a numbertrack or number line and count on in ones to find the answer Bar model encourages children to count on rather than counting all	Place the larger number in your head and count on the smaller number to find your answer. $12 + 5 = 17$ Continue to use missing numbers in all possible places. $\square = 16 + 7$ $16 + \square = 23$
Regrouping to make 10	Start with the bigger number and use the smaller number to add up to make 10. Use ten frames, dienes, numicon $6 + 5 = 11$ $10 + 1 = 11$ $6 + 6 = 12$ $10 + 2 = 12$ $9 + 4 = 13$	Use pictures and regroup the numbers to make 10 Use a numberline to partition the smaller number to make a 10	Record method of adding by counting up to 10 $8 + 7 =$ $10 + 5 = 15$ Consolidate as missing number problems $8 + 5 = 10 + \square$ $\square + 7 = 10 + 4$
Adding more than two 1-digit numbers	Look for bonds to 10 then add extra digits	Regroup and draw representation. $4 + 7 + 6 = 17$	$4 + 7 + 6 = 10 + 7$ Combine the two numbers that make/bridge ten then add on the third.
Adding with 2 digit numbers and bridging 10s	Use a 10 frame to Make up to the next 10 $17 + 5 = 22$ Use dienes or multilink to make whole 10s.	Bridge through a multiple of 10. Use part-part-whole and number line to model adding to the next whole 10 Children should be able to partition the 7 to relate adding the 2 and then the 5. $8 + 7 = 15$ Adding 9 or 11 by adding 10 and adjusting	Record method of bridging to 10 $17 + 5 = 20 + 3 = 23$ Record how to add near 10s then adjust $35 + 9$ $35 + 10 = 45$ $45 - 1 = 44$ Explore related facts $17 + 5 = 22$ $5 + 17 = 22$ Bar model can be used to support missing number problems $10 + 12 =$ $11 + \square = 25$

Adding with no regrouping (carrying): working towards column method	Add Units (ones) together first then Tens. Model using dienes, numicon, place value counters or multilink 24 + 15  45 34 7 9 44 + 15 = 59  Arrow cards can be used to support understanding of place value 140 + 232 = 100 + 40 + 0 + 200 + 30 + 2 = 300 + 70 + 2 	Use partitioning to add Units (ones) and Tens separately then recombine 27 + 32  Tens: 20 + 30 = 50 Ones: 7 + 2 = 9 50 + 9 = 59 Children can use place value columns to record methods 41 + 8 = 49 	Progress to addition in columns securing understanding of columns representing place value Ensure children are using 1 digit per box Always add from the right: Units first, then Tens, then Hundreds. 75 + 3 = 78 11 + 11 = 22 2 2 3 + 1 1 4 ----- 3 3 7
Adding with regrouping (carrying): working towards expanded column method	Use dienes or multilink to show partitioning and recombining grouping Units (ones) and Tens 47 + 25 = 60 + 12  Leading to carrying: 72 Children begin to understand the exchange between hundreds, tens and ones. 36 + 25 	Use written methods of partitioning and recombining Tens and Units, then Hundreds, Tens and Units 60 + 12 25 + 26 40 + 10 = 50 40 + 7 = 47 20 + 5 = 25 60 + 12 = 72 369 + 148 	Children apply partition understanding by working in the units column first, then the Tens column etc. 67 + 24 = 47 + 25 = 72 60 + 12 = 72 75 + 3 = 78 Secure understanding of the place value of each column. 3 6 4 + 2 7 8 ----- 6 4 2
Adding with regrouping (carrying): working towards collapsed column method	Place value counters or multilink can be used to represent: carrying ten '1s' over to make one '10' carrying ten '10s' over to make one '100' 243 + 368 	Place value diagrams can support understanding of carrying into the next column   Place value grids can be used to develop understanding of addition with decimal numbers. 	Column addition can now be done using the collapsed method, carrying Tens and Hundreds directly over into the next column. 247 + 125 = 372 5,647 + 2,994 = 8,641 183 + 46 = 229 Carried numbers should be placed at the TOP Children should start adding decimals in known contexts such as measures or money £3.75 + £2.53 = £6.28 Progress to adding larger decimals including those with different numbers of decimal places. 382.46 + 237.25 = 619.71 Example: 0.32 + 12.965 + 1.1 = 14.385 Example: 51 + 14.02 + 2.1 = 67.12

	Concrete	Pictorial	Abstract
Taking away by counting back	Use physical objects, counters, cubes etc to show how objects can be taken away Subtraction with numbers 1 to 5	Cross out drawn objects to show what has been taken away. Count back in ones using a number line.	Put 13 in your head, count back 4. What number are you at? $7 - 4 = 3$ Include calculations either side of the equality sign and missing number calculations $__ = 12 - 8$ $9 - __ = 5$
Find the difference	Compare objects and amounts '7 is 3 more than 4' '4 is 3 less than 7' 'The difference between 4 and 7 is 3' Use objects to represent bar model	Use pictures or draw objects to represent questions of difference and how many more or less. How many more pencils than erasers? What is the difference between 8 and 5? Bar model can also be used Progress to counting on using a number line Find the difference between 5 and 11	Hannah has 12 sweets and her sister has 5. How many more does Hannah have than her sister? $5 + __ = 12$ Put 5 in your head and count on to 12 Progress to an understanding of calculating difference between by subtracting the smaller number from the larger number. $12 - 5 = 7$ Children begin to appreciate this as the inverse to counting on or adding $5 + 7 = 12$ $12 - 5 = 7$
Use number bonds up to 20	Link to addition - use Part Part Whole model If 10 is the whole and 6 is one parts, what is the other part? $10 - 6 = 4$	Use pictorial representations $6 - 2 = __$ $7 - 3 = __$	Move to using numbers with the part whole model. Use part whole or bar method to support constructing number sentences and interpreting word problems $14 - 5 = ?$ $5 + ? = 14$
Counting back to the nearest 10	14 - 9 Make 14 on the ten frame. Take 4 away to make ten, then take one more away so that you have taken 5. Numicon can be used.	14 - 5 Show pictorially counting 4 back to the last 10 then counting 1 more back 13 - 7 Jump back 3 first, then another 4. Use the ten as a stopping point. $13 - 7 = 6$	Show how to count back to the next 10 by partitioning the subtrahend $14 - 5 = 9$ $14 - 4 = 10$ $10 - 1 = 9$ Mental methods: $16 - 8$ How many do we take off first to get to 10? How many more to take off?

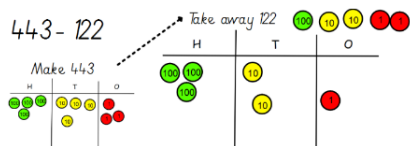
Use dienes or PV counters to show how to partition the number and take away from each column, starting with the Units (ones)

$$75 - 45 = 30$$

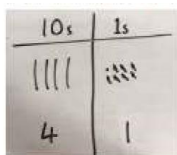


$$\begin{array}{r} 70 \ 5 \\ - 40 \ 5 \\ \hline 30 \ 0 \end{array}$$

$$443 - 122 = 321$$



Children draw representations to cross off



$$43 - 21 = 22$$

$$48 - 7 = 41$$

Understand numbers can be partitioned before subtracting:

$$\begin{array}{r} 47 - 24 = 23 \\ - 20 + 7 \\ \hline 20 + 3 \end{array}$$

$$846 - 275$$

$$\begin{array}{r} 800 \ 40 \ 6 \\ - 200 \ 70 \ 5 \\ \hline \end{array}$$

Subtract in columns

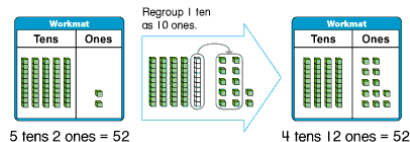
Always start subtracting from the smallest place value (right hand end)

$$\begin{array}{r} 48 \\ - 7 \\ \hline 41 \end{array}$$

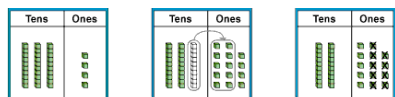
$$\begin{array}{r} 28 \\ - 12 \\ \hline 16 \end{array}$$

Highlight importance of 1 digit per box and correct place value for each column.

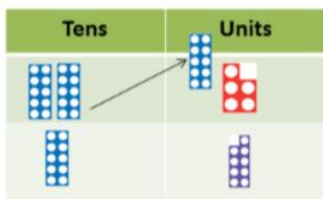
Use numicon, dienes, PV counters or multilink to demonstrate physically exchanging 1 ten for 10 units (ones) before subtracting.



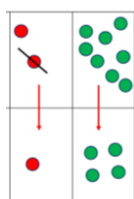
$$34 - 9 = 25$$



$$35 - 19 =$$

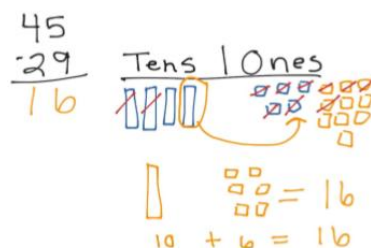


$$32 - 14 = 18$$

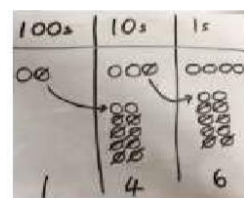


NB: Use language of exchanging 1 ten for 10 Units (ones) not borrowing

Children might draw base ten or PV counters to show exchanging and cross off as they take away.



$$234 - 88 + 146$$



Formal column method.

Children should understand that they exchanged 1 from the Tens column for 10 in the Units (ones) column.

They still have 41 because $30 + 11 = 41$

$$\begin{array}{r} 41 \\ - 26 \\ \hline 15 \end{array}$$

Note the red numbers showing what has been exchanged are placed just to the left of the calculation digits NOT above

Progress to larger numbers and exchanging over multiple columns.

$$\begin{array}{r} 254 \\ - 1562 \\ \hline 1192 \end{array}$$

Introduce decimal subtraction in the context of money or measures

$$£3.80 - £1.26$$

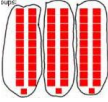
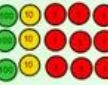
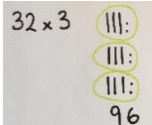
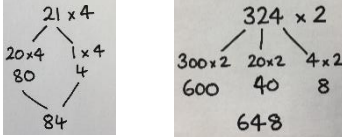
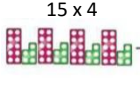
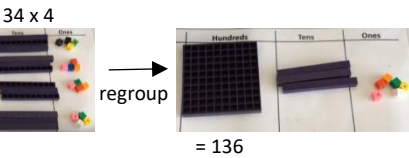
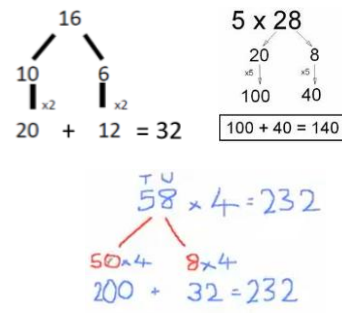
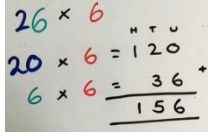
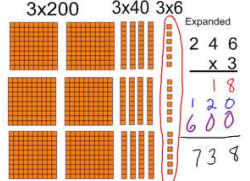
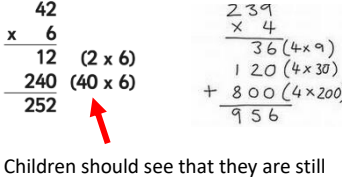
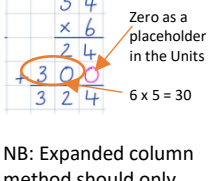
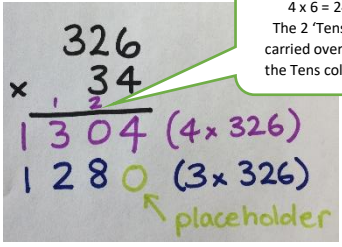
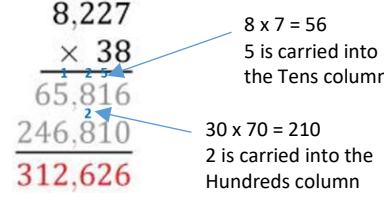
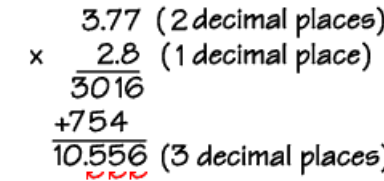
$$\begin{array}{r} 3.80 \\ - 1.26 \\ \hline 2.54 \end{array}$$

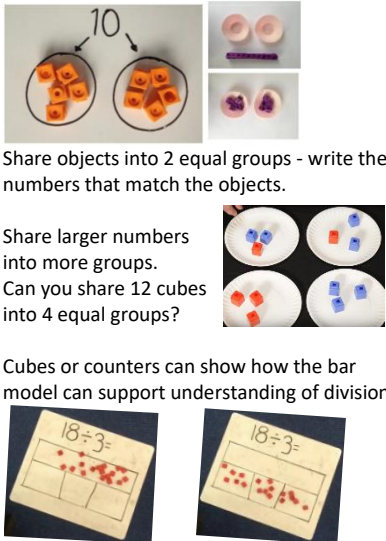
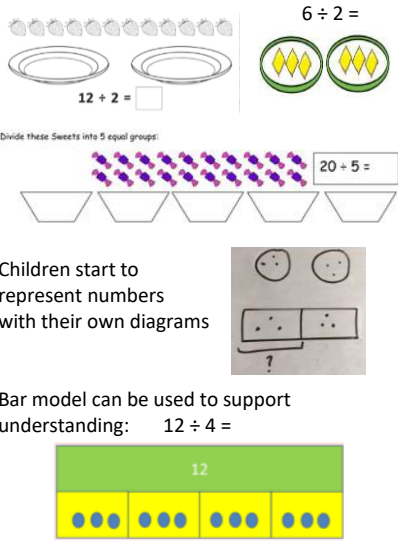
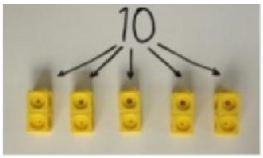
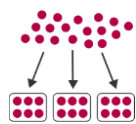
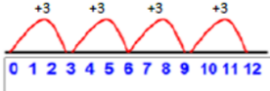
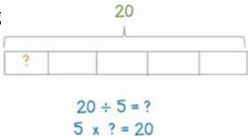
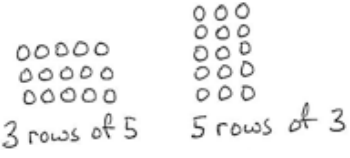
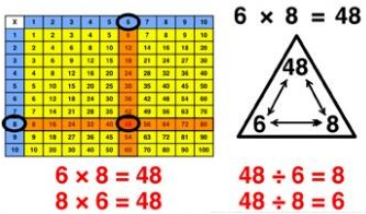
Use larger decimal numbers including zeros for place holders

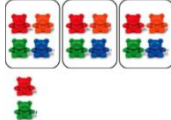
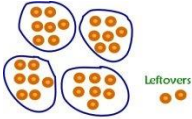
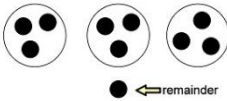
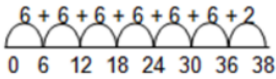
$$\begin{array}{r} 7.680 \\ - 3.725 \\ \hline 3.955 \end{array}$$

$$\begin{array}{r} 5.349 \\ - 3.608 \\ \hline 1.741 \end{array}$$

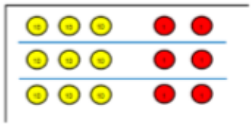
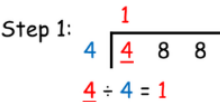
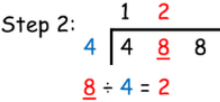
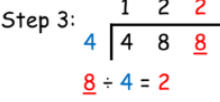
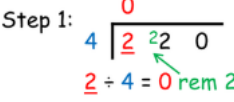
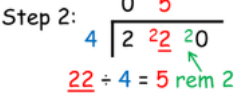
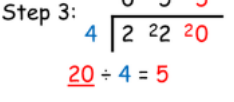
	Concrete	Pictorial	Abstract
Doubling	Use practical activities using manipulatives including objects, cubes and Numicon to demonstrate doubling Double 4 = 8	Use or draw pictures or symbols to show doubling numbers What is double 3? Double 4 is 8 What is double 4? $4 + 4 = 8$	Double numbers by understanding 'adding the number to itself' and/or '2 lots of' (multiply by 2) What is double 3? $3 + 3 = 6$ OR $3 \times 2 = 6$
Counting in multiples	Skip counting using objects, multilink, representations 1, 2, 3, 4, 5, 6	Skip counting using a numberline Children use representations to show counting in groups	Count in multiples of a number aloud. Use songs and rhymes. Write sequences with multiples of numbers by counting on. 3, 6, 9, 12... Missing numbers 5 10 15 25
Repeated addition	Use different groups of objects to add equal groups $3 + 3 + 3$	Use pictures of equal groups to model repeated addition $4 + 4 + 4 = 12$ There are 3 sweets in one bag. How many sweets are in 5 bags altogether? $3 + 3 + 3 + 3 + 3 = 15$	Progress to numberlines showing repeated jumps Repeated addition without numberline $4 + 4 + 4 = 12$
Using Arrays	Use objects / multilink laid out in arrays using the language 'lots of' : 3 lots of 3 3×3 5 lots of 7 5×7 Use arrays to illustrate commutativity 4 groups of 3 3 groups of 4	Show arrays using pictures and grouping into rows or columns 3 lots of 3 3×3 4 lots of 3 4×3 3 lots of 2 3×2 Children represent arrays using dots $3 \times 5 = 15$ Continue to use arrays to show commutativity	Use arrays to write multiplications sentences and reinforce repeated addition. $5 + 5 + 5 = 15$ $3 + 3 + 3 + 3 + 3 = 15$ $5 \times 3 = 15$ $3 \times 5 = 15$ Write abstract number sentences: $4 \times 6 = 24$ $6 \times 4 = 24$
Doubling: No regrouping	Use dienes, multilink, Place Value cards/counters. Partition to double each digit and recombine Double 12 2 lots of 10 2 lots of 2 = 24 Double 122 2 lots of 100 2 lots of 10 2 lots of 2 = 224	Children can draw diagrams to support doubling Progress to partitioning into Tens and Units. Show doubling each digit then adding together. Reinforce that doubling means multiplying by 2	Doubling mentally using Place Value.

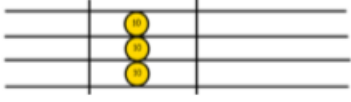
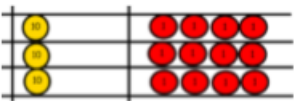
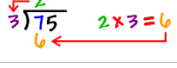
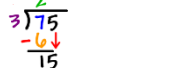
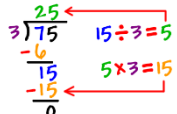
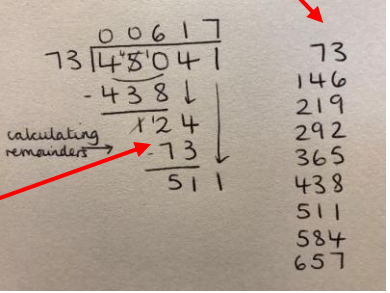
multiplying by partitioning: No regrouping	Use concrete resources to lay out 'lots of' each digit. Multiply each digit then add to recombine.  $21 \times 3 = 63$ $3 \text{ lots of } 20 = 60$ $3 \text{ lots of } 2 = 6$  $113 \times 3 = 339$ $3 \text{ lots of } 100 = 300$ $3 \text{ lots of } 10 = 30$ $3 \text{ lots of } 3 = 9$ NB: At this stage, children should notice how to add simply by looking at Place Value not by counting on.	Children can represent Tens and Units (Ones) to help multiply  Partition the 2 or 3 digit number. Multiply each digit then recombine. 	Partition into Hundreds, Tens, Units (Ones). Multiply each digit then recombine. $431 \times 2 = (400 \times 2) + (30 \times 2) + (1 \times 2)$ $800 + 60 + 2 = 862$
Partitioning to multiply: with regrouping (carrying)	Children can continue to be supported by dienes, Numicon or place value counters to show regrouping to carry over into Tens/Hundreds  $15 \times 4 = 60$ $10 \times 4 = 40$ $5 \times 4 = 20$  $34 \times 4 = 136$	Use diagrams to show partitioning and multiplying each digit then adding the products. 	Partition numbers mentally and show each multiplication separately in brackets. $23 \times 4 = (20 \times 4) + (3 \times 4)$ $80 + 12 = 92$ $5 \times 123 = (5 \times 100) + (5 \times 20) + (5 \times 3)$ $500 + 100 + 15 = 615$ Column addition could be used to support addition of larger numbers. 
Column multiplication: Expanded method	Dienes or Place Value counters can be used as manipulatives to support understanding of the column method if needed.  NB: We do not use any grid multiplication methods	Set out calculations with one digit per box, checking Place Value is aligned correctly. Multiply Units first.  Children should see that they are still partitioning the larger number and initially be shown each multiplication in brackets.	This should lead to an understanding of using zeros as a placeholders and working with single digit multiplications  NB: Expanded column method should only be used for multiplying by a single digit.
Column multiplication: Collapsed (compact) method	Abstract: x single digit Each digit in the multiplier is multiplied by the multiplicand (5), starting with the Units. multiplication for each digit is now not shown on a separate line.  When needed, number are carried over into the next column using smaller digits placed above the answer	Abstract: x double digits Children should be confident with the collapsed method before attempting multiplication by double digits. The expanded method should not generally be used for multiplication by a double digit number. The multiplier is multiplied by each digit in the multiplicand (34) starting with the Units.  A placeholder is needed in the Units column for the second row of calculations as we are multiplying by 30, not just 3.	Abstract: x decimals Make sure all 'carried' digits are smaller than the numbers in the main calculation and placed above the immediate answers so they are not missed.  Decimals should not be aligned for column multiplication. The calculation is completed in the same way as for whole numbers. The total number of decimal places in the question should equal the number of decimal places in the answer.  Children should understand this as adjusting the answer by dividing it by multiples of 10.

	Concrete	Pictorial	Abstract
Division as sharing	Use a range of language to help children understand division: sharing/splitting/dividing into x equal groups  Share objects into 2 equal groups - write the numbers that match the objects. Share larger numbers into more groups. Can you share 12 cubes into 4 equal groups? Cubes or counters can show how the bar model can support understanding of division.	Children use pictures or shapes to share into equal groups. Start finding half – link with 'dividing by 2' and use ÷ symbol.  Children start to represent numbers with their own diagrams Bar model can be used to support understanding: $12 \div 4 =$	Children can use their fingers or counting out loud to support mental strategies to share numbers. Start by halving. Describe as dividing by 2. $12 \div 2 = 6$ Use word problems: I have 16 sweets and share them equally between 2 children. How many sweets will each child get? $16 \div 2 = 8$ Divide with larger divisors: What is 12 shared between 3? 4 $9 \div 3 = 3$
Division as grouping	Dividing quantities into equal groups $10 \div 2$  Sort 10 objects / counters / multilink into groups of 2. How many groups of 2 can you make? 5 Group counters along a numberline. $20 \div 5$  How many groups of 5 do you need to get to 20?	Show grouping using pictures or diagrams of objects. $18 \div 6$ How many groups of 6 can you make with 18?  $27 \div 3$ Children draw their own diagrams. How many groups of 3 can you make? Numberlines can be used or drawn to show grouping with digits. $12 \div 3$ How many jumps of 3 to make 12?  Bar Model: think of the bar as the whole number. Split it into the number of groups you are dividing by and work out how many would be in each g 	$24 \div 6 = 4$ Ask: How many groups of 6 are there in 24? How many times does 6 'go into' 24? Children should be encouraged to find answers by counting up in 'groups of' the divisor. They should start to use times tables knowledge or times tables grids for support.
Division with arrays	Link division to multiplication by making an array and thinking about the number sentences that can be created $5 \times 3 = 15$ $3 \times 5 = 15$ $15 \div 3 = 5$ $15 \div 5 = 3$ 	Draw an array and split the number in different ways, showing the multiplication and division calculations that match.  $5 \times 3 = 15$ $3 \times 5 = 15$ $15 \div 3 = 5$ $15 \div 5 = 3$	Explore division as the inverse of multiplication $3 \times 7 = 21$ Has the inverse relationships $21 \div 3 = 7$ $21 \div 7 = 3$ Reinforce commutativity by writing fact families  $6 \times 8 = 48$ $8 \times 6 = 48$ $48 \div 6 = 8$ $48 \div 8 = 6$

Division with remainders	Divide objects, counters, cubes by grouping and see how many are leftover. $14 \div 4$  There are 3 groups of 4, with 2 leftover $28 \div 7 =$ 4 with 2 leftover 	Children use or draw their own diagrams to show groups of the divisor. Introduce 'remainder' for the number leftover. $10 \div 3$  $38 \div 6$ Ask 'How many 6s are there in 38?' Use a numberline to count up in jumps of 6 and identify the remainder.  = 6 sixes with a remainder of 2	Complete written divisions and show remainder using 'r' $28 \div 5$ Ask: how many 5s can you get in 28? Count up in 5s. Question understanding: Can you go as far as $5 \times 5 = 25$? $5 \times 6 = 30$? Why not? Show that they stop at the last multiple before the dividend and then check how many more to count on. $5 \times 5 = 25$ count on 3 to get to 28 $28 \div 5 = 5 \text{ r}3$
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NB: It is important that children have a sound understanding of abstract division and are confident counting up in times tables mentally before attempting formal written methods. Most children will not need concrete or pictorial support at this stage. Multiplication grids should be used to support children less confident with times tables

Towards standard written methods: Short Division (Bus Stop method)	Concrete resources to support standard written methods: Place Value counters or Dienes could be used to aid understanding of dividing each digit in turn starting with the largest Place Value. $96 \div 3$ Tens Units 3 2  Show alongside bus stop method layout. These representations can be used to demonstrate the need to exchange any remainders in the tens for units and then divide the units equally.	Begin with numbers which divide equally and digits do not need to be exchanged and moved over to the next column. To work out $488 \div 4$: Step 1:  Step 2:  Step 3: 	Next tackle numbers where one or more digit within the calculation needs to be exchanged and moved to the next column but the calculation has no overall remainder. To work out $220 \div 4$: Step 1:  Step 2:  Step 3: 	Progress to divisions where exchanging is needed and the calculation has a remainder. Show how remainders could be expressed as a decimal or fraction. $573 \div 4 = 13$  How many 4's in 5? 1 r1 How many 4's in 17? 4 r1 How many 4's in 13? 3 r1 0.355 $4 \overline{)142.0}$ Write zeros in decimal places as needed for exchanging remainders 0.7125 $8 \overline{)57.000}$ 0.663 remainder 5 $8 \overline{)53.09}$ divisor 8
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Standard written methods: Long Division	$42 \div 3 =$ Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.   We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14.	It is usually only necessary to use long division when dividing by double digits. However, some children may benefit from using this method with smaller numbers if they are struggling to understand bus stop method. Divide:  Multiply:  Subtract:  Bring Down:  Repeat:  Give reminders that children may need to use exchanging in the subtraction calculations as they find remainders. NB: We do not use a 'chunking' method $11 \text{ r} 1$ $25 \overline{)276}$ Look to see how many times 25 can be divided by 25! Bring down the number to the right! Subtract that product! Repeat the process until there are no more numbers to bring down! 1. Divide 2. Multiply 3. Subtract 4. Bring Down Children should be encouraged to write multiples of the divisor alongside their calculation 
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